

Lecture 2: Equity Premium

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- Some basic facts
- Study the asset pricing implications of household portfolio choice
- Consider the quantitative implications of a second-order approximation to asset return equations
- Reference: Mehra and Prescott (JME, 1984)

UNITED STATES ANNUAL DATA, 1889-1978

- average real consumption growth rate 1.8% per year
- real consumption growth rate standard deviation 2%
- average real equity return 8% per year
- average real riskless rate 1% per year (US Tbill - Inflation)
- equity premium in the data is large 7% per year on an annual basis
- Question: Can a simple model generate this equity premium?

Mehra and Prescott Economy

- Two assets only (simplification of Lucas economy) and a representative household
- One asset is risk free and pays return R_t^f known at time t
- The other asset is risky and pays return R_t , unknown at time t realized at time $t + 1$.
- Representative household makes portfolio choices, chooses to invest quantity B_t in the riskless asset, and quantity S_t in the risky asset, to maximize

$$E_t \sum_{s=0}^{\infty} \beta^s u(c_{t+s}), \quad 0 < \beta < 1$$

- subject to the budget constraint

$$B_{t+1} + S_{t+1} + c_t = R_t^f B_t + R_t S_t$$

$$B_{t+1} : u'(c_t) = R_{t+1}^f \beta E_t [u'(c_{t+1})]$$

$$S_{t+1} : u'(c_t) = \beta E_t [u'(c_{t+1}) R_{t+1}]$$

- Rearranging we have:

$$1 = R_{t+1}^f E_t \left[\frac{\beta u'(c_{t+1})}{u'(c_t)} \right]$$

$$1 = E_t \left[\frac{\beta u'(c_{t+1})}{u'(c_t)} R_{t+1} \right]$$

- $u'(c_t) = \text{constant}$
- The pricing equations become

$$R_{t+1}^f = E_t R_{t+1}$$

- Pricing equation implies

$$E_t [m_{t+1} R_{t+1}] = 1$$

where m_{t+1} is discount factor or pricing kernel and R_{t+1} is the return.

- The pricing equation implies the discount factor depends on consumption:

$$m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$$

Implications (1)

- For stocks

$$P_t = E_t \{ m_{t+1} X_{t+1} \}$$

where $X_{t+1} = (D_{t+1} + P_{t+1})$ and $R_{t+1} = \frac{D_{t+1} + P_{t+1}}{P_t}$

- Recursive substitution

$$P_t = E_t \{ m_{t+1} (D_{t+1} + E_{t+1} \{ m_{t+2} (D_{t+2} + P_{t+2}) \}) \}$$

- With risk neutrality $m_{t+s} = \beta^s$ and using the law iterated expectations

$$P_t = E_t \left\{ \sum_{s=0}^{\infty} \beta^s D_{t+s} \right\}$$

Implications (2)

- Let $\beta = 1/(1+r)$ and assume D_t is random walk with drift

$$D_t = (1+g) D_{t-1} + \varepsilon_t,$$

where ε_t is white noise with mean 0 and variance σ_ε .

- Then

$$E_t D_{t+s} = (1+g)^s D_t, \text{ all } t$$

- This implies

$$P_t = \sum_{s=0}^{\infty} \left(\frac{1+g}{1+r} \right)^s D_t$$

- Assume $\frac{1+g}{1+r} < 1$. Then:

$$\left(1 - \frac{1+g}{1+r} \right) P_t = D_t$$

or

$$\frac{P_t}{D_t} = \frac{r-g}{1+r}$$

Implications (3)

- For risk-free one-period bond that pays one unit of consumption tomorrow:

$$P_t = E_t \{ m_{t+1} \}$$

where

$$R_{t+1} = \frac{1}{P_t}$$

- Nominal securities:

$$1 = E_t \left\{ m_{t+1} \frac{X_{t+1}^n}{P_t^n} \frac{1}{1 + \pi_{t+1}} \right\}$$

where

$$R_{t+1}^n = \frac{X_{t+1}^n}{P_t^n}$$

is the nominal return and

$$1 + \pi_{t+1} = \frac{P_{t+1}^{CPI}}{P_t^{CPI}}$$

Consumption-Based Asset Pricing (1)

- Using pricing equation for risky and riskless assets:

$$E_t \left[\frac{\beta u'(c_{t+1})}{u'(c_t)} R_{t+1} \right] = R_{t+1}^f E_t \left[\frac{\beta u'(c_{t+1})}{u'(c_t)} \right]$$

using the covariance formula

$$\text{cov}(x, y) = E [(x - E_x)(y - E_y)] = E(xy) - E_x E_y$$

$$\text{cov}_t(m_{t+1}, R_{t+1}) + E_t m_{t+1} E_t R_{t+1} = R_{t+1}^f E_t m_{t+1}$$

and rearranging gives

$$\left(E_t R_{t+1} - R_{t+1}^f \right) E_t m_{t+1} = -\text{cov}_t(m_{t+1}, R_{t+1})$$

Consumption-Based Asset Pricing (2)

- Using pricing equation for riskless asset

$$E_t m_{t+1} = \frac{1}{R_{t+1}^f}$$

in the equation

$$\left(E_t R_{t+1} - R_{t+1}^f \right) E_t m_{t+1} = -\text{cov}_t (m_{t+1}, R_{t+1})$$

get

$$\frac{E_t R_{t+1} - R_{t+1}^f}{R_{t+1}^f} = -\text{cov}_t (m_{t+1}, R_{t+1})$$

Consumption-Based Asset Pricing (3)

- If the risky return covaries positively with tomorrow's consumption, c_{t+1} , then the LHS is positive and the asset return bears a positive premium over the risk free rate.
- If the risky return covaries negatively with tomorrow's consumption then the LHS is negative and the asset return bears a negative premium over the risk free rate.
- Intuition: assets whose returns have a negative covariance with consumption provide a hedge against consumption risk. Households are willing to accept a lower expected return since these assets provide insurance against low future consumption.

The equity premium puzzle

- Assume

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$$

- The Euler equations are

$$c_t^{-\gamma} = R_{t+1}^f E_t \beta c_{t+1}^{-\gamma}$$

$$c_t^{-\gamma} = E_t \left[R_{t+1} \beta c_{t+1}^{-\gamma} \right]$$

Approximation to the Euler equation

- Let $x_{t+1} = \ln(c_{t+1}) - \ln(c_t)$, $r_{t+1} = \ln(R_{t+1})$, the Euler equations become:

$$1 = \beta E_t \exp(-\gamma x_{t+1} + r_{t+1}^f)$$

$$1 = \beta E_t (-\gamma x_{t+1} + r_{t+1})$$

- Assume that consumption growth and asset returns are jointly log-normally distributed:

$$\begin{bmatrix} x_{t+1} \\ r_{t+1} \end{bmatrix} \sim N \left(\begin{bmatrix} \bar{x}_{t+1} \\ \bar{r}_{t+1} \end{bmatrix}, \begin{bmatrix} \text{var}(x_{t+1}) & \text{cov}(x_{t+1}, r_{t+1}) \\ \text{cov}(x_{t+1}, r_{t+1}) & \text{var}(r_{t+1}) \end{bmatrix} \right)$$

Remember that if $x \sim N(\bar{x}, \sigma_x^2)$ then $X = \exp(x)$ is log-normal distributed with

$$E(X) = \exp\left(\bar{x} + \frac{1}{2}\sigma_x^2\right)$$

- The Euler equations become

$$1 = \beta \exp \left(-\gamma \bar{x}_{t+1} + r_{t+1}^f + \frac{1}{2} \text{var}(-\gamma x_{t+1}) \right)$$

$$1 = \beta \exp \left(-\gamma \bar{x}_{t+1} + \bar{r}_{t+1} + \frac{1}{2} \text{var}(-\gamma x_{t+1} + r_{t+1}) \right)$$

Implications (1)

- Take logs and equate these equations:

$$-\gamma \bar{x}_{t+1} + r_{t+1}^f + \frac{1}{2} \text{var}(-\gamma x_{t+1}) = -\gamma \bar{x}_{t+1} + \bar{r}_{t+1} + \frac{1}{2} \text{var}(-\gamma x_{t+1} + r_{t+1})$$

$$\begin{aligned} \bar{r}_{t+1} - r_{t+1}^f &= \frac{1}{2} [\text{var}(-\gamma x_{t+1}) - \text{var}(-\gamma x_{t+1} + r_{t+1})] \\ &= -\frac{1}{2} \text{var}(r_{t+1}) + \gamma \text{cov}(x_{t+1}, r_{t+1}) \end{aligned}$$

where we used the formula

$$\text{var}(y + z) = \text{var}(y) + \text{var}(z) + 2\text{cov}(y, z)$$

Implications (2)

- As $R_{t+1} = \log r_{t+1}$ then $\log E_t R_{t+1} = \bar{r}_{t+1} + \frac{1}{2} \text{var}(r_{t+1})$ and replace in

$$\bar{r}_{t+1} - r_{t+1}^f = -\frac{1}{2} \text{var}(r_{t+1}) + \gamma \text{cov}_t(x_{t+1}, r_{t+1})$$

to get

$$\log E_t R_{t+1} - \log R_{t+1}^f = \gamma \text{corr}_t(x_{t+1}, r_{t+1}) \sigma_{x_{t+1}} \sigma_{r_{t+1}}$$

where we used the formula

$$\text{corr}_t(x_{t+1}, r_{t+1}) = \text{cov}_t(x_{t+1}, r_{t+1}) / (\sigma_{x_{t+1}} \sigma_{r_{t+1}})$$

- The equity premium is:

$$\log E_t R_{t+1} - \log R_{t+1}^f = \gamma \text{corr}(x_{t+1}, r_{t+1}) \sigma_{x_{t+1}} \sigma_{r_{t+1}}$$

- In US data, $\sigma_r = 0.167$, $\sigma_x = 0.036$, $\text{corr}(x, r) = 0.4$ so
 - If $\gamma = 1$ we have $\log E_t R_{t+1} - \log R_{t+1}^f = 0.24\%$.
 - If $\gamma = 10$ we have $\log E_t R_{t+1} - \log R_{t+1}^f = 2.4\%$.
 - If $\gamma = 25$ we have $\log E_t R_{t+1} - \log R_{t+1}^f = 6.0\%$.

too high? (1)

Example 1. What would be the interest rate that would make a household that earns 50,000 euros per year willing to postpone an annual vacation that costs 3,000 euros?

$$R_{t+1}^f = \beta E_t \left(\frac{c_{t+1}}{c_t} \right)^\gamma$$

taking $\beta = 1$, $\gamma = 25$ and no uncertainty in the income process get

$$R_{t+1}^f = \left[\frac{\beta u'(c_{t+1})}{u'(c_t)} \right]^{-1} = \left(\frac{53,000}{47,000} \right)^{25} = 20.16 \approx 2016\%$$

too high? (2)

Example 2. The Certainty Equivalent, CE , of a lottery that gives 50,000 euros with 50% probability or 100,000 euros with 50% probability

$$\frac{(CE)^{1-\gamma}}{1-\gamma} = \frac{1}{2} \frac{(50,000)^{1-\gamma}}{1-\gamma} + \frac{1}{2} \frac{(100,000)^{1-\gamma}}{1-\gamma}$$

$$\gamma = 0 \quad CE = 75,000$$

$$\gamma = 1 \quad CE = 70,711$$

$$\gamma = 2 \quad CE = 66,246$$

$$\gamma = 5 \quad CE = 58,566$$

$$\gamma = 10 \quad CE = 53,991$$

$$\gamma = 20 \quad CE = 51,858$$

$$\gamma = 30 \quad CE = 51,209$$

Quantitative implications: Risk free rate

- From Euler equation

$$1 = \beta \exp \left(-\gamma \bar{x}_{t+1} + r_{t+1}^f + \frac{1}{2} \text{var}(-\gamma x_{t+1}) \right)$$

get the risk free rate is:

$$r_{t+1}^f = -\log \beta + \gamma \bar{x}_{t+1} - \frac{\gamma^2}{2} \text{var}(x_{t+1})$$

- Suppose $\beta = 0.999$ (in order for r_{t+1}^f to be as small as possible), $x = 0.015$ (tx. cresc. cons.), $\sigma_x = 0.036$ then we need $\gamma = 0.6$ to get $r_{t+1}^f = 1\%$.
 - If $\gamma = 10$ get $r_{t+1}^f = 22\%$
 - If $\gamma = 25$ get $r_{t+1}^f = 78\%$
- This is opposite to equity-premium puzzle – we need very low γ to match risk-free rate

Additional quantitative implications: Risk free rate

- If consumption growth is iid and homoskedastic, then risk free rate is constant.
- Risk free rate is higher if consumers are more impatient (have a high relative preference for consumption in the present) i.e. have a low β (time preference)
- Risk free rate is higher when expected consumption growth (\bar{x}_{t+1}) is higher (intertemporal substitution).
- Risk free rate is low when conditional consumption volatility ($var(c_{t+1})$) is high (precautionary savings).

Conclusions:

1. The simple model cannot explain the level of the equity premium.
2. It was first formalized by Mehra and Prescott in 1984. It remains a mystery: the difference is too large to reflect a "proper" level of compensation that would occur as a result of investor risk aversion
3. Possible explanations:
 - (i) A large differential in the cost of trading between the stock and bond markets;
 - (ii) more general preferences that allow for the separation between risk aversion and IMRS;
 - (iii) incomplete markets (representative agent),
 - (iv) borrowing constraints,
 - (v) market segmentation (heterogeneity of agents).

Observation:

With power utility function the γ controls both:

1. The **intertemporal marginal rate of substitution** (changes of consumption across time due to changes in the relative price of consumption today vs future)
2. **Risk aversion** (changes of consumption across states of nature, which applies only when there is uncertainty).
3. **Precautionary savings** has to do with the volatility of consumption, and is related with the 3rd derivative of the utility function

More facts on the equity premium (1)

- **Fact 1:** Risk premiums also vary over time, with a clear business-cycle correlation. Stock, bond, and currency returns can be forecasted by regressions of the form

$$R_{t+1}^e = a + by_t + \varepsilon_{t+1}$$

using as the forecasting variable y_t the price/dividend or price/earnings ratio of stocks, yield spreads of bonds, or interest rate spreads across countries.

- Expected returns vary over time as much as their level: $\sigma(E_t(R_{t+1}^e)) = \sigma(a + by_t)$ is large compared to $E(R^e)$: If the equity premium is 6% on average, it is as likely to be 1% or 11% at any moment in time. (A regression of returns on dividend yields gives a standard error of expected returns $\sigma(E_t(R_{t+1}^e)) = \sigma(a + by_t)$ of 5.5 percentage points.)

More facts on the equity premium (2)

- **Fact 2:** Furthermore, expected returns are high, prices are low, and risk premiums are high, in a coordinated way across many asset classes, in the bottoms of recessions. Conversely, expected returns are low, prices are high, and risk premiums are low at the tops of booms.
- **Problem:** need a model that can explain simultaneously a high equity premium and a higher equity premium in bad times.

How to fix the model?

To explain these facts, the macro-finance literature explored a wide range of alternative preferences and market structures.

1. Habits (Campbell and Cochrane, 1999).
2. Recursive utility (Epstein and Zin, 1989).
3. Long run risks (Bansal and Yaron, 2004; Bansal, Kiku, and Yaron, 2012).
4. Idiosyncratic risk (Constantinides and Duffie, 1996).
5. Heterogeneous preferences (Gârleanu and Panageas, 2015).
6. Rare Disasters (Reitz, 1988; Barro, 2006).
7. Utility nonseparable across goods (Piazzesi, Schneider, and Tuzel, 2007).
8. Leverage; balance-sheet; (Brunnermeier, 2009; Krishnamurthy and He, 2013).
9. Ambiguity aversion, min-max preferences, (Hansen and Sargent, 2011).
10. Behavioral finance; probability mistakes (Shiller, 1981, 2014).

Intuition (1)

- These approaches look different, but in the end the ideas are quite similar. The strategies in the literature boil down to generalize the discount factor to

$$m_{t+1} = \frac{\beta u'(c_{t+1})}{u'(c_t)} Y_{t+1}$$

where the new variable Y_{t+1} does most of the work, or allow the probability of future states to depend on the current state in a particular way - higher probability of a future disaster if the current state is bad.

Intuition (2)

- In the habit model, endogenous time-varying individual risk aversion is at work - people are less willing to take risks in bad times.
- In long-run risks, rare disasters and idiosyncratic risks models, the risk itself is time-varying.
- In heterogeneous agent models, the market has a time-varying risk-bearing capacity, though neither risks, or individual risk aversion, or individual probability mis-perceptions need vary over time.
- In heterogeneous agent models, changes in the wealth distribution that favor more or less risk averse agents induces the shift in risk-bearing capacity.